

# Combustion Theory

*Second Edition*

**Forman A. Williams**

**ABP**

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Forrest A. Williams  
Princeton University

The Fundamental Theory of  
Chemically Reacting Flow Systems

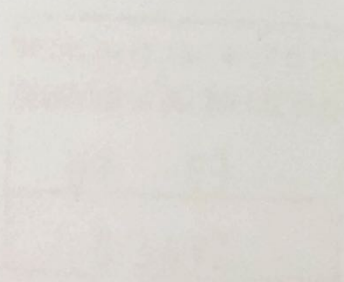
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the leader of combustion  
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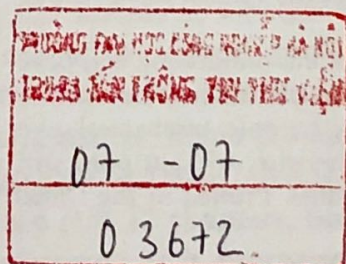
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Chemically Reacting Flow Systems*

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**Forman A. Williams**

*Princeton University*



The Advanced Book Program

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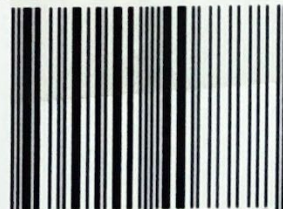
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