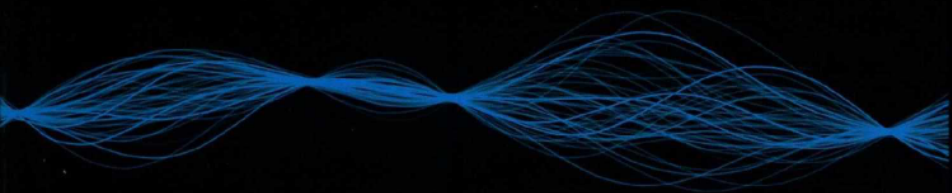


# Algorithms for Optimization



Mykel J. Kochenderfer and Tim A. Wheeler



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*To our families.*





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