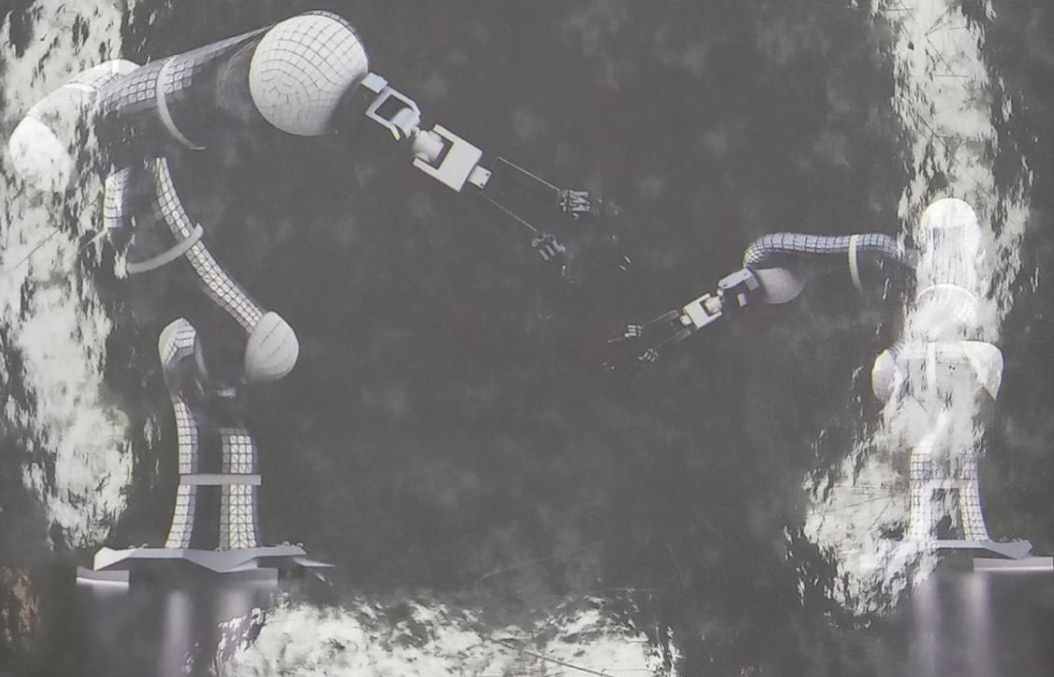


MARK W. SPONG | SETH HUTCHINSON  
M. VIDYASAGAR

# ROBOT MODELING AND CONTROL

SECOND EDITION



WILEY

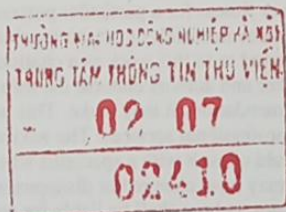
# Robot Modeling and Control

Second Edition

Mark W. Spong

Seth Hutchinson

M. Vidyasagar



WILEY

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# Preface

This text is a second edition of our book, *Robot Modeling and Control*, John Wiley & Sons, Inc., 2006, which grew out of the earlier text, M.W. Spong and M. Vidyasagar, *Robot Dynamics and Control*, John Wiley & Sons, Inc., 1989. The second edition reflects some of the changes that have occurred in robotics and robotics education in the past decade. In particular, many courses are now treating mobile robots on an equal footing with robot manipulators. As a result, we have expanded the discussion on mobile robots into a full chapter. In addition, we have added a new chapter on under-actuated robots. We have also revised the material on vision, vision-based control, and motion planning to reflect changes in those topics.

## Organization of the Text

After the introductory first chapter, which introduces the terminology and history of robotics and discusses the most common robot design and applications, the text is organized into four parts. Part I consists of four chapters dealing with the geometry of rigid motions and the kinematics of manipulators.

Chapter 2 presents the mathematics of rigid motions; rotations, translations, and homogeneous transformations.

Chapter 3 presents solutions to the forward kinematics problem using the Denavit–Hartenberg representation, which gives a very straightforward and systematic way to describe the forward kinematics of manipulators.

Chapter 4 discusses velocity kinematics and the manipulator Jacobian. The geometric Jacobian is derived in the cross product form. We also introduce the so-called analytical Jacobian for later use in task space control. We have reversed the order of our treatment of velocity kinematics and inverse kinematics from the presentation in the first edition in order to include a new section in Chapter 5 on numerical inverse kinematics algorithms, which rely on the Jacobian for their implementation.

Chapter 5 deals with the inverse kinematics problem using the geometric

approach, which is especially suited for manipulators with spherical wrists. We show how to solve the inverse kinematics in closed form for the most common manipulator designs. We also discuss numerical search algorithms for solving inverse kinematics. Numerical algorithms are increasingly popular because of both the increasing power of computers and the availability of open-source software for numerical algorithms.

Part II deals with dynamics and motion planning and consists of two chapters.

Chapter 6 is a detailed account of robot dynamics. The Euler–Lagrange equations are derived from first principles and their structural properties are discussed in detail. The recursive Newton–Euler formulation of robot dynamics is also presented.

Chapter 7 is an introduction to the problems of path and trajectory planning. Several of the most popular methods for motion planning and obstacle avoidance are presented, including the method of artificial potential fields, randomized algorithms, and probabilistic roadmap methods. The problem of trajectory generation is presented as essentially a problem of polynomial spline interpolation. Trajectory generation based on cubic and quintic polynomials as well as trapezoidal velocity trajectories are derived for interpolation in joint space.

Part III deals with the control of manipulators.

Chapter 8 is an introduction to independent joint control. Linear models and linear control methods based on PD, PID, and state space methods are presented for set-point regulation, trajectory tracking, and disturbance rejection. The concept of feedforward control, including the method of computed torque control, is introduced as a method for nonlinear disturbance rejection and for tracking of time-varying reference trajectories.

Chapter 9 discusses nonlinear and multivariable control. This chapter summarizes much of the research in robot control that took place in the late 1980s and early 1990s. Simple derivations of the most common robust and adaptive control algorithms are presented that prepare the reader for the extensive literature in robot control.

Chapter 10 treats the force control problem. Both impedance control and hybrid control are discussed. We also present the lesser known hybrid impedance control method, which allows one to control impedance and regulate motion and force at the same time. To our knowledge this is the first textbook that discusses the hybrid impedance control approach to robot force control.

Chapter 11 is an introduction to visual servo control, which is the problem of controlling robots using feedback from cameras mounted either on

the robot or in the workspace. We present those aspects of vision that are most useful for vision-based control applications, such as imaging geometry and feature extraction. We then develop the differential kinematics that relate camera motion to changes in extracted features and we discuss the main concepts in visual servo control.

Chapter 12 is a tutorial overview of geometric nonlinear control and the method of feedback linearization of nonlinear systems. Feedback linearization generalizes the methods of computed torque and inverse dynamics control that are covered in Chapters 8 and 9. We derive and prove the necessary and sufficient conditions for local feedback linearization of single-input/single-output nonlinear systems, which we then apply to the flexible joint control problem. We also introduce the notion of nonlinear observers with output injection.

Part IV is a completely new addition to the second edition and treats the control problems for underactuated robots and nonholonomic systems.

Chapter 13 deals with underactuated serial-link robots. Underactuation arises in applications such as bipedal locomotion and gymnastic robots. In fact, the flexible-joint robot models presented in Chapters 8 and 12 are also examples of underactuated robots. We present the ideas of partial feedback linearization and transformation to normal forms, which are useful for controller design. We also discuss energy and passivity methods to control this class of systems.

Chapter 14 deals primarily with wheeled mobile robots, which are examples of systems subject to nonholonomic constraints. Many of the control design methods presented in the chapters leading up to Chapter 14 do not apply to nonholonomic systems. Thus, we cover some new techniques applicable to these systems. We present two fundamental results, namely Chow's theorem and Brockett's theorem, that provide conditions for controllability and stabilizability, respectively, of mobile robots.

Finally, the appendices have been expanded to give much of the necessary background mathematics to be able to follow the development of the concepts in the text.

## A Note to the Instructor

This text is suitable for several quarter-long or semester-long courses in robotics, either as a two- or three- course sequence or as stand-alone courses. The first five chapters can be used for a junior/senior-level introduction to robotics for students with at least a minimal background in linear algebra. Chapter 8 may also be included in an introductory course for students with some exposure to linear control systems. The independent joint control

problem largely involves the control of actuator and drive-train dynamics; hence most of the subject can be taught without prior knowledge of Euler-Lagrange dynamics.

A graduate-level course on robot dynamics and control can be taught using all or parts of Chapters 6 through 12.

Finally, one or more special topics courses can be taught using Chapters 9 through 14. Below we outline several possible courses that can be taught from this book:

### **Course 1: Introduction to Robotics**

Level: Junior/Senior undergraduate

For a one quarter course (10 weeks):

**Chapter 1:** Introduction

**Chapter 2:** Rigid Motions and Homogeneous Transformations

**Chapter 3:** Forward Kinematics

**Chapter 4:** Velocity Kinematics and Jacobians

**Chapter 5:** Inverse Kinematics

For a one semester course (16 weeks) add:

**Chapter 7:** Motion Planning and Trajectory Generation

**Chapter 8:** Independent Joint Control

### **Course 2: Robot Dynamics and Control**

Level: Senior undergraduate/graduate

For a one quarter course (10 weeks):

**Chapters 1–5:** Rapid Review of Kinematics (selected sections)

**Chapter 6:** Dynamics

**Chapter 7:** Path and Trajectory Planning

**Chapter 9:** Nonlinear and Multivariable Control

**Chapter 10:** Force Control

For a one semester course (16 weeks) add:

**Chapter 11:** Vision-Based Control

**Chapter 12:** Feedback Linearization

### **Course 3: Advanced Topics in Robot Control**

Level: Graduate

For a one semester course (16 weeks):

**Chapter 6:** Dynamics

**Chapter 7:** Motion Planning and Trajectory Generation

**Chapter 9:** Nonlinear and Multivariable Control

**Chapter 11:** Vision-Based Control

**Chapter 12:** Feedback Linearization

**Chapter 13: Underactuated Robots****Chapter 14: Mobile Robots**

The instructor may wish to supplement the material in any of these courses with additional material to delve deeper into a particular topic. Also, either of the last two chapters can be covered in Course 2 by eliminating the Force Control chapter or the Vision-Based Control chapter.

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Mark W. Spong  
Seth Hutchinson  
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